A Qualitative Vickrey Auction

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Vickrey versus Qualitative Vickrey

Vickrey’s sealed-bid second-price single item auction
- bids are prices
- outcome: winner has highest bid, price of second-highest bid
- bidding private value is a dominant strategy

Qualitative Vickrey auction
- bids are alternatives
- outcome: winner has highest ranked bid, alternative at least as high as second-highest
- bidding highest acceptable alternative is a dominant strategy
Motivating Example: Buy a Super-computer

Limited budget (e.g. from a project) to buy a super-computer

1. Announce ranking of alternatives (including budget) to suppliers
2. Request one (sealed) proposal from each supplier
3. Select winner: supplier with most preferred proposal
4. Select deal (by supplier): higher preferred than second-ranked proposal
Outline

1 Definitions
   • Notation and Definitions
   • The Qualitative Vickrey Auction
   • Adequate Strategies

2 Properties
   • Dominant Strategies
   • Pareto Efficiency
   • Other Properties

3 Summary and Future Work
   • Summary
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Definitions and Assumptions

Notation and Definitions

- An outcome is an alternative and a winner: \((a, i) \in A \times N\).
- Center’s order over \(A \times N\) is given by a linear order \(\geq\).
- Bidder \(i\)’s preferences over \(A \times N\) are given by a weak order \(\succsim_i\).

Assumptions

- Bidder \(i\) can only bid from \(A \times \{i\}\).
- Bidder \(i\) is indifferent between outcomes where winner is not \(i\).
- Assume each bidder has at least one acceptable outcome, where an outcome \((a, i)\) is acceptable to \(i\) if \((a, i) \succsim_i (x, j)\) for \(j \neq i\).
The Qualitative Vickrey Auction

The *qualitative Vickrey auction* follows the following protocol:

1. The order $\succeq$ of the center is publicly announced.
2. Each bidder $i$ submits a sealed bid $(a, i) \in A \times \{i\}$.
3. The bidder $i^*$ who submitted the bid ranked highest in $\succeq$ is the winner.
4. The winner $i^*$ may choose from $A \times \{i^*\}$ any outcome ranked at least as high as *second-highest* bid in $\succeq$. 
Example of a Qualitative Vickrey Auction

\[(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \cdots > (c,1) > \cdots > (d,3)\]
Example of a Qualitative Vickrey Auction

\[(a, 1) > (a, 2) > (a, 3) > (b, 1) > (b, 2) > \cdots > (c, 1) > \cdots > (d, 3)\]
Adequate Strategies

A strategy for $i$ is adequate if

1. $i$ bids acceptable outcome ranked highest in $\geq$, and
2. if $i$ wins the auction, $i$ selects outcome she prefers most (in $\succ_i$) from those ranked higher in $\geq$ than the second-highest bid.
**Example of Using an Adequate Strategy**

\[(a, 1) > (a, 2) > (a, 3) > (b, 1) > (b, 2) > \cdots > (c, 1) > \cdots > (d, 3)\]

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### Example of Using an Adequate Strategy

\[(a, 1) \succ (a, 2) \succ (a, 3) \succ (b, 1) \succ (b, 2) \succ \cdots \succ (c, 1) \succ \cdots \succ (d, 3)\]

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Example of Using an Adequate Strategy

\[(a, 1) > (a, 2) > (a, 3) > (b, 1) > (b, 2) > \cdots > (c, 1) > \cdots > (d, 3)\]

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Adequate Strategies are Dominant

Theorem

*Adequate strategies are dominant.*

Proof.

(sketch)

- Let \((a, i)\) be acceptable outcome (to \(i\)) ranked highest in \(\geq\).
- Let \((a', j)\) be highest-ranked bid by \(j \neq i\).
- Two cases:
  1. \((a', j) > (a, i)\): \(i\) should bid below \((a', j)\) in \(\geq\), because if \(i\) wins, she can only select unacceptable outcomes, and
  2. \((a, i) > (a', j)\): \(i\) should bid above \((a', j)\) in \(\geq\), because then outcome can be highest in \(\succsim_i\), which is above \((a', j)\).
- In both cases, optimal strategy for \(i\) is to bid \((a, i)\).
DSE is Not Strongly Pareto Efficient

\[(a, 1) > (a, 2) > (a, 3) > (b, 1) > (b, 2) > \cdots > (c, 1) > \cdots > (d, 3)\]

\[
\begin{array}{ccc}
\sim_1 & \sim_2 & \sim_3 \\
(b, 1) & (b, 2) & (d, 3) \\
(x, i) \notin A \times \{1\} & (x, i) \notin A \times \{2\} & (a, 3) \\
\vdots & \vdots & (x, i) \notin A \times \{3\} \\
\end{array}
\]

Bidder 3 will win with outcome \((a, 3)\), while

1. \((d, 3)\) is strictly higher preferred by bidder 3, and
2. all other bidders are indifferent.
The dominant strategy equilibrium is

- Weakly Pareto efficient: no outcome is *strictly* preferred by *all* bidders.
- Strongly Pareto efficient when center is also considered: other outcome is either worse for center, or for winner.
- Weakly monotonic: if a bidder moves the equilibrium outcome \((a^*, i^*)\) up in its order, the outcome of the mechanism stays the same.
Summary

- A class of auctions without money, similar to Vickrey’s second-price auction
- A dominant strategy equilibrium that is
  - weakly Pareto efficient (but not strongly),
  - strongly Pareto efficient when center is also considered, and
  - weakly monotonic.
- In paper:
  - Escape Gibbard-Satterthwaite by restricting bidders’ preferences (distinct acceptable outcomes and indifferent among non-winning)
  - Drop assumption that each bidder has an acceptable outcome
Future Work

- Prove that the Vickrey auction with money is a special case (where $\geq$ is the standard order over prices)
- Show relation to multi-attribute auctions
- Study other qualitative auctions (e.g. English, multi-unit, online)
- Characterise instances of these mechanisms (parameterised by $\geq$)
- Find more interesting applications without money transfers (e.g. grids)