DICHOTOMOUS COLLECTIVE DECISION-MAKING

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OUTLINE OF THE COURSE

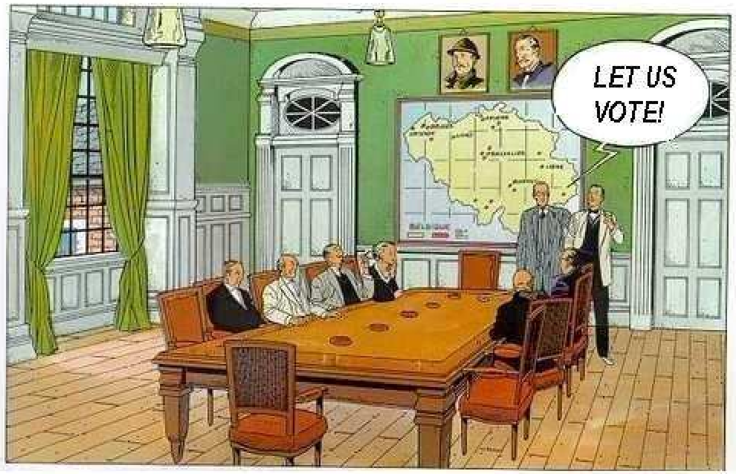
I. Introduction

II. Binary dichotomous voting rules

III. Ternary-Quaternary dichotomous voting rules

INTRODUCTION

SIMPLEST VOTING SITUATION



A TWO-THIRD MAJORITY IS NECESSARY TO ELECT HIM

An external proposal is submitted to the committee

The members of the committee vote (yes/no)

The proposal is accepted or not

INTRODUCTION: STUDIED SITUATIONS

- Situation where a group of people have to make decide on accept or reject a proposal with the help of a voting rule
- Examples: Parliament, Council, Jury, Referendum,…

- Assumptions
  - Binary choice: yes – no
  - Dichotomous final decision: accepted – rejected

INTRODUCTION: ADDRESSED QUESTIONS

- How easy is it to adopt proposals?
  - Simple majority versus unanimity versus dictatorship
  - The answer depends on the voting rule.
  - If voters independently vote yes with proba ½ versus if voters independently vote yes with proba 1/5
  - The answer depends on the voting behavior

INGREDIENTS OF THE MODELS

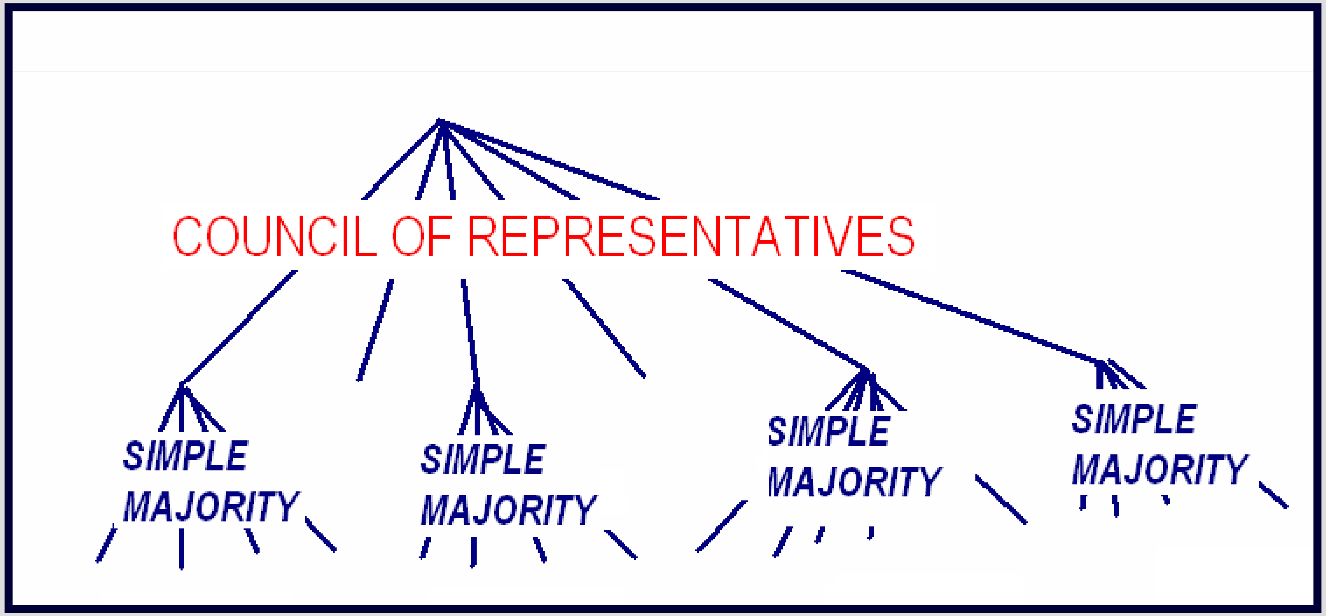
- Voting rule
- Voting behaviour

INTRODUCTION: ADDRESSED QUESTIONS

- From a normative point of view, what is the best rule?
  - Normative: all configurations equally probable
  - Egalitarianism: equal utility for all voters
  - Utilitarianism: to maximize the sum of utilities
  - Utility obtained by a voter: associate a level of utility to the four possible outcome:
    - The voter has voted yes and the proposal is accepted
    - The voter has voted yes and the proposal is rejected
    - The voter has voted no and the proposal is rejected
    - The voter has voted no and the proposal is accepted

INTRODUCTION: ADDRESSED QUESTIONS

- What is the **most adequate** voting rule for a committee if each member acts on behalf of a group of individuals or a constituency of different sizes?



INTRODUCTION

- In Parliament the rules used are more complex. In particular they are not binary
  - Simple majorities with participation quorum
  - Majority of present voters
- How to model these more complex rules?

OUTLINE OF THE COURSE

I. Introduction

II. Binary voting rules

A. Model

i. Voting rules

ii. Voting behaviour

B. Ease to pass proposal

C. Best voting rules

D. Application to the European Union

MODEL - VOTING RULE : DEFINITIONS

Let us consider a rule with n seats.

$N = \{1, 2, .., n\}$, set of labels.

2^n possible configurations of votes

$S \subset N$, *vote configuration* $S = \{i \,|i \text{ votes yes}\}$

VOTING RULE

S is winning if it leads to the passage of the proposal.

W denotes the set of winning configurations

$$W = \{S \mid S \text{ leads to a final 'yes'}\} .$$

MODEL - VOTING RULES: PROPERTIES

W denotes the set of winning configurations

1. $N \in W$
2. $\emptyset \notin W$
3. If $S \in W$, then $T \in W$ for any T containing S
4. If $S \in W$ then $N \backslash S \notin W$

Remark No possible manipulation: a voter always follows her or his preferences

MODEL - VOTING RULES: EXAMPLES

- Simple Majority

Simple majority

$$W^{SM} = \{S \mid s > \frac{n}{2}\}.$$

Symmetric rule

- k-Majority (k>1/2)

k-majority rule

$$W^{kM} = \{S \mid s \geq kn\}.$$

Symmetric rule

- Weighted Majority

Weighted majority

$$W^{(w,Q)} = \{S \subseteq N : \sum_{i \in S} w_i \geq Q\}.$$

Non Symmetric rule

MODEL - VOTING RULES: EXAMPLES

- Dictatorship

Dictatorship

$$W^{D_i} = \{S \subseteq N : i \in S\}$$

- Seat i has a veto

$$i \notin S \Rightarrow S \notin W$$

- Oligarchy

T – Oligarchy

$$\mathcal{W}^T = \{S \subseteq N : S \supseteq T\}.$$

Non Symmetric rules

- Unanimity

Unanimity

$$\mathcal{W}^N = \{N\}.$$

Symmetric rule

MODEL - VOTING RULES: REMARKS

- In a dictatorship the dictator will always get his or her preferred outcome.
- Whenever a voter has a veto right, he or she will always get his or her preferred outcome whe he or she votes no.
- It is more difficult to pass a proposal with unanimity than with a simple majority
- Is it more easy to adopt a proposal under the {1,2}-oligarchy than under the {1,3}-oligarchy?

MODEL - VOTING BEHAVIOUR: DEFINITION

$$
\begin{aligned}
\text{Map } p &: 2^N \rightarrow R \\
p(S) &= \text{probability that } S \text{ emerges} \\
&= \text{probability that voters in } S \text{ vote'yes'} \\
&\quad \text{and voters in } N \backslash S \text{ vote 'no'.}
\end{aligned}
$$

$$
0 \leq p(S) \leq 1 \text{ for any } S \subseteq N \text{ and } \sum_{S \subseteq N} p(S) = 1
$$

MODEL - VOTING BEHAVIOUR: EXAMPLES

- Voters vote independently of each others

$$p^{(t_1,\ldots,t_n)}(S) = \prod_{i \in S} t_i \prod_{j \in N \setminus S} (1 - t_j).$$

3 voters, each voter independently votes from the others,

- the first one votes with probability 1/2 'yes',
- the second has a probability 1/8 to vote 'yes' and
- the third one a probability 1/4 to vote 'yes'.

MODEL - VOTING BEHAVIOUR: EXAMPLES

- 4 voters
  - The first three voters voter independently, they vote 'yes' with probability 1/2.
  - The fourth voter follows the majority of the other three voters.

MODEL - NORMATIVE VOTING BEHAVIOUR

- FOR A NORMATIVE APPROACH

  Behind a veil of ignorance: all vote configurations have the same probability:

$$p^*(S) = \frac{1}{2^n}$$

Equivalently: All voters independently vote 'yes' and 'no' with probability 1/2

$$P(i \in S) = P(i \notin S) = \frac{1}{2} \quad \text{for all } i$$

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I. Introduction

II. **Binary voting rules**

A. Model

i. Voting rules

ii. Voting behaviour

B. Ease to pass proposal

C. Best voting rules

D. Application to the European Union

EASE TO PASS PROPOSALS: DEFINITION

- It is more difficult to pass a proposal with unanimity than with a simple majority
- Is it more easy to adopt a proposal under the {1,2}-oligarchy than under the {1,3}-oligarchy?
  - It depends on p
- A measure of the easiness to adopt proposals: Probability that a proposal is adopted:

$$\alpha(\mathcal{W}, p) := Prob\ \{\text{acceptance}\} = \sum_{S: S \in \mathcal{W}} p(S),$$

EASE TO PASS PROPOSALS: PROPERTIES

- Property

If $\mathcal{W} \subseteq \mathcal{W}'$, then for any p,

$$\alpha(\mathcal{W}, p) \leq \alpha(\mathcal{W}', p),$$

- It is more difficult to pass a proposal with unanimity than with a simple majority

  W={{1,2,3}} and W'={{1,2},{1,3}, {2,3}, {1,2,3}}
- Is it more easy to adopt a proposal under the {1,2}-oligarchy than under the {1,3}-oligarchy?

  W= '={{1,2},{1,2,3}} and W'={{1,3}, {1,2,3}}

EASE TO PASS PROPOSALS: NORMATIVE

- Positive evaluation versus normative evaluation
  - Positive evaluation: p as close as possible to the real data
  - Normative evaluation p*

$$p^*(S) = \frac{1}{2^n}$$

$$\alpha(W, p^*) = Prob\,\{\text{acceptance}\} = \sum_{S: S \in W} p^*(S)$$

OUTLINE OF THE COURSE

I. Introduction

II. Binary voting rules

A. Model

B. Ease to pass proposal

C. Best voting rules

i. Egalitarianism

ii. Utilitarianism

iii. In direct committees

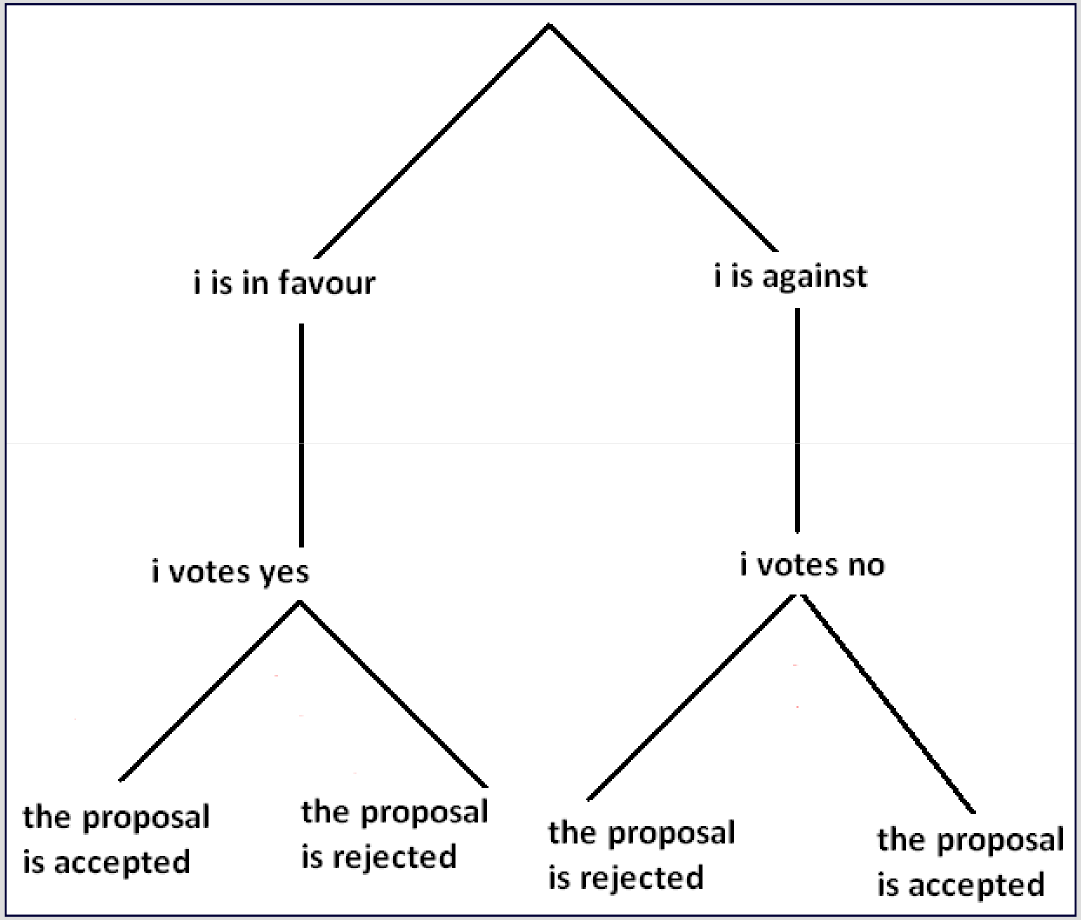
iv. In indirect committees

MOST ADEQUATE VOTING RULE?

- From a normative point of view, what is the best rule?
  - Egalitarianism: equal utility for all voters
  - Utilitarianism: to maximize the sum of utilities

Define the utility obtained by a voter

VOTER i'S UTILITY FOR A GIVEN ISSUE



$$\begin{aligned} u_{i+}(Acc) &= A^{i+} \\ u_{i+}(Rej) &= R^{i+} \\ u_{i-}(Acc) &= A^{i-} \\ u_{i-}(Rej) &= R^{i-} \end{aligned}$$

VOTER i'S UTILITY FOR ANY ISSUE

Assumptions:

Symmetry among issues

Symmetry among voters

$$u_i(\mathcal{W}, S) = \begin{cases} A^+ \text{ if } i \in S \in \mathcal{W}, \\ R^+ \text{ if } i \in S \notin \mathcal{W}, \\ R^- \text{ if } i \notin S \notin \mathcal{W}, \\ A^- \text{ if } i \notin S \in \mathcal{W}, \end{cases}$$

Define

Proposal Accepted (A) — A^+

$$\Delta^+ := A^+ - R^+ > 0$$

Proposal Rejected (R) — R^+

R^- — Proposal Rejected (R)

$$\Delta^- := R^- - A^- > 0$$

A^- — Proposal Accepted (A)

VOTER i'S UTILITY FOR A RULE

$$E_p\left[u_i(\mathcal{W},S)\right] = A^+P(i \in S \in \mathcal{W}) + R^+P(i \in S \notin \mathcal{W})$$
$$+A^-P(i \notin S \in \mathcal{W}) + R^-P(i \notin S \notin \mathcal{W}),$$

NORMATIVE APPROACH

$$p^*(S) = \frac{1}{2^n}$$

$$P(i \in S \in \mathcal{W}) = \sum_{S: i \in S \in \mathcal{W}} \frac{1}{2^n}$$

etc

BEST VOTING RULE?

EGALITARIANISM: choose the rule (W) in order to get

$$E_p\left[u_i(\mathcal{W},S)\right] = E_p\left[u_j(\mathcal{W},S)\right], \text{ for all } i,j.$$

UTILITARIANISM: choose the rule (W) in order to

$$Max \sum_{i \in N} E_p\left[u_i(\mathcal{W},S)\right].$$

BEST VOTING RULE? EGALITARIANISM

EGALITARIANISM : choose the rule (W) in order to get

$$E_p\left[u_i(\mathcal{W},S)\right] = E_p\left[u_j(\mathcal{W},S)\right], \text{ for all } i,j.$$

Any symmetric rule satisfies egalitarianism

k-majority rule

$$W^{kM} = \{S \mid s \geq kn\}.$$

In particular the simple majority, the unanimity

BEST VOTING RULE? UTILITARIANISM

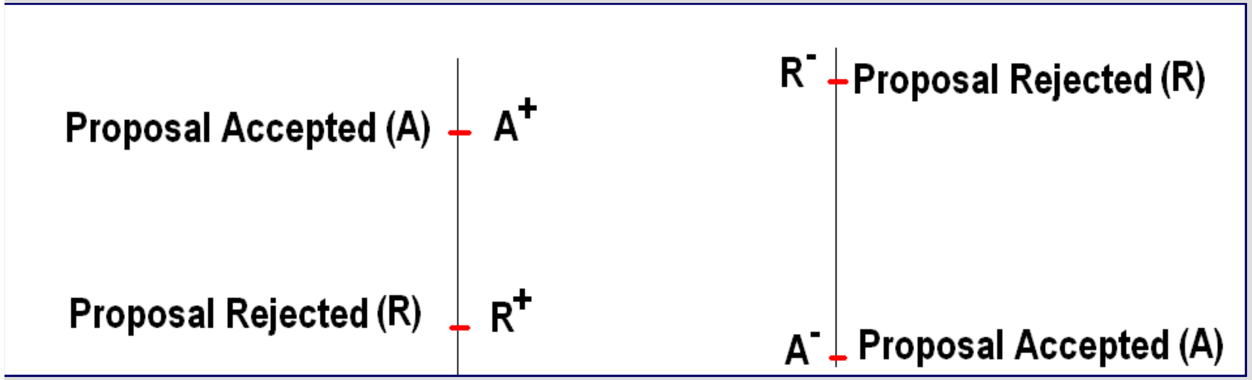
Choose the rule (W) in order to $Max \sum_{i \in N} E_p\left[u_i(\mathcal{W}, S)\right].$

The result depends on whether

$\Delta^- \geq \Delta^+$ or $\Delta^- < \Delta^+$

Recall $\Delta^+ := A^+ - R^+ > 0$

$\Delta^- := R^- - A^- > 0$



$\Delta^- \geq \Delta^+$ means:

it is more important

to get a rejection

when against

than

to get an acceptance

when in favour

BEST VOTING RULE? UTILITARIANISM

Choose the rule (W) in order to

$$Max \sum_{i \in N} E_p \left[u_i(\mathcal{W}, S) \right].$$

If $\Delta^- \geq \Delta^+$

the k-majority rule implements the utilitarian principle with k= $\dfrac{\Delta^-}{\Delta^+ + \Delta^-}$

If $\Delta^- < \Delta^+$

the simple majority rule implements the utilitarian principle when the number of voters is odd.

BEST VOTING RULE? UTILITARIANISM

Interpretation:

- If the same importance is given to obtaining the preferred outcome with a acceptance or a rejection, then the best rule is the simple majority
- If more importance is given to obtaining the preferred result with a rejection then $k>1/2$ (extreme case: unanimity, $k=1$)
- If more importance is given to obtaining the preferred result with a acceptance then as $k<1/2$ impossible $k=1/2$

BEST VOTING RULE

- Direct committees

  Both principles can be satisfied at once:

  - Egalitarianism: choose any k-majority rule
  - Utilitarianism: choose a k-majority rule with k = $\frac{\Delta^-}{\Delta^+ + \Delta^-}$

- Indirect committees?

Example: EU Council of Ministers

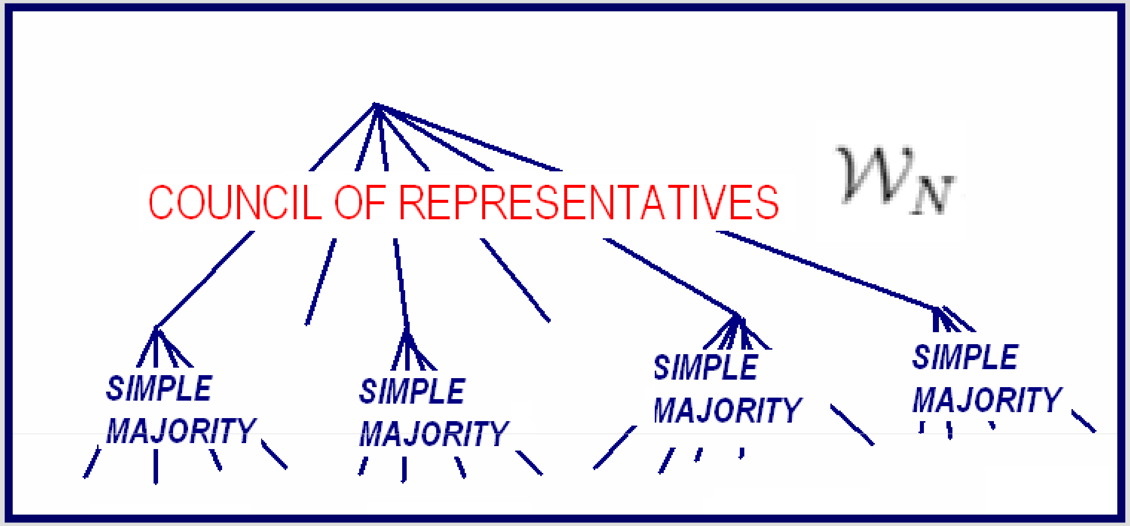
BEST VOTING RULE IN INDIRECT COMMITTEES

Indirect Committee or Committees of representatives

- Data:
  - number of members in the committee
  - sizes of each group represented

- Question
  - Which rule should be used in the Committee?

MODEL OF INDIRECT COMMITTEES



$$S_N = \{i \in N : \#S_i > \frac{m_i}{2}\}$$

$$S_i = S \cap M_i.$$

$$\mathcal{W}_M = \{S \subseteq M : S_N \in \mathcal{W}_N\}.$$

- Assumption: representatives follow the majority opinion of his/her group on every issue

INDIRECT COMMITTEES: EGALITARIANISM

EGALITARIANISM : choose the rule in the committee in order to get equal expected utilities among citizens

$$E_p\left[u_k(\mathcal{W}_M, S_M)\right] = E_p\left[u_l(\mathcal{W}_M, S_M)\right] \text{ for all } k, l \in M.$$

- Assumption: citizens behave independently (p=p*)

Choose the rule in the Committee such that

$$\frac{1}{\sqrt{m_i}} \sum_{\substack{S: i \in S \in \mathcal{W} \\ S \setminus i \notin \mathcal{W}}} \frac{1}{2^{n-1}} = \frac{1}{\sqrt{m_j}} \sum_{\substack{S: j \in S \in \mathcal{W} \\ S \setminus j \notin \mathcal{W}}} \frac{1}{2^{n-1}} \text{for any } i, j \in N$$

in practice any rule will do in the EU (mi and mj large)

INDIRECT COMMITTEES: UTILITARIANISM

UTILITARIANISM: choose the rule in order to

$$Max \sum_{i \in N} \sum_{k \in M_i} E_p \left[u_k(\mathcal{W}_M, S_M) \right].$$

- Weight = Square root rules of the size of the represented group ($\sqrt{m_i}$)

- Quota $Q(\frac{\Delta^-}{\Delta^+}) = \frac{1}{2} \sum_{i \in N} \sqrt{m_i} + \frac{1}{2} \frac{\frac{\Delta^-}{\Delta^+} - 1}{\frac{\Delta^-}{\Delta^+} + 1} m \sqrt{\frac{\pi}{2}}$

Similar to direct committees: Q increases with $\frac{\Delta^-}{\Delta^+}$

BEST VOTING RULE: SUMMARY

- Direct committees
  - Egalitarianism: choose a k-majority rule
  - Utilitarianism: k-majority rule with $k = \Delta^{-} / (\Delta^{+} + \Delta^{-})$
- Committees of representatives
  - Egalitarianism: any rule
  - Utilitarianism: weighted majority
    - Weight = Square root of the represented group
    - Quota = $Q(\Delta^{+} / \Delta^{-})$

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III. Ternary and quaternary voting rules

APPLICATION TO THE EUROPEAN UNION

COUNCIL OF MINISTERS VOTING RULES

Simple Majority ($\mathcal{W}^{SM}$)

$$\mathcal{W}^{SM} = \left\{ S \subseteq N : s > \frac{n}{2} \right\}$$

Unanimity ($\mathcal{W}^{U}$)

$$\mathcal{W}^{U} = \{N\}$$

Qualified Majority ($\mathcal{W}^{QM}$)

$$\mathcal{W}^{QM} = \left\{ S \subseteq N : \sum_{i \in S} w_i(N) \geq Q(N) \right\}$$

WEIGHTS AND QUOTA IN THE QUALIFIED MAJORITY

N_6={**Ge, Fr, It, Ne, Be, Lu**}; w_6= {**4, 4, 4, 2, 2, 1**}, Q_6= **12**

N_9={Ge, **UK**, Fr, It, Ne, Be, **De, Ir**, Lu};

w_9= {10, **10**, 10, 10, 5, 5, **3, 3**, 2}, Q_9= **41**

N_{10}={Ge, UK, Fr, It, Ne, **Gr**, Be, De, Ir, Lu};

w_{10}= {10, 10, 10,10, 5, **5**, 5, 3, 3, 2}, Q_{10}= **45**

N_{12}= {Ge, UK, Fr, It, **Sp**, Ne, Gr, Be, **Pr**, De, Ir, Lu};

w_{12}= {10, 10, 10,10, **8**, 5, 5, 5, **5**, 3, 3, 2}, Q_{12}= **54**

N_{15}= {Ge, UK, Fr, It, Sp, Ne, Gr, Be, Pr, **Sw, Au**, De, **Fi**, Ir, Lu};

w_{15}= {10, 10, 10,10, 8, 5, 5, 5, 5, **4, 4**, 3, **3**, 3, 2}, Q_{15}= **62**

HOW EASY IS IT TO PASS A PROPOSAL IN THE EU?

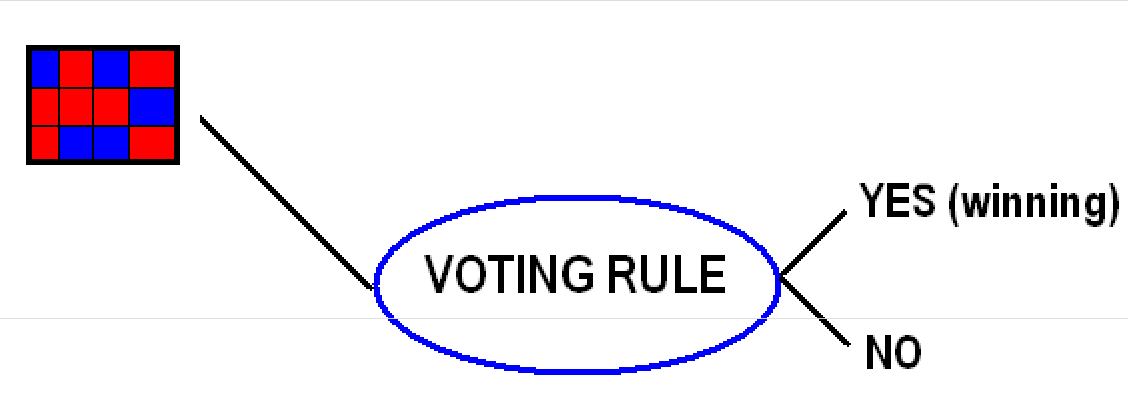
	N_6	N_9	N_{10}	N_{12}	N_{15}
$\alpha(\mathcal{W}^{SM}, p^*)$	0.344	0.5	0.377	0.387	0,5
$\alpha(\mathcal{W}^{U}, p^*)$	0.016	0.002	0.001	0.0002	0.00003
$\alpha(\mathcal{W}^{QM}, p^*)$	0.219	0.146	0.137	0.098	0.078

$$\alpha(\mathcal{W}_N^U, p_N^*) \quad < \quad \alpha(\mathcal{W}_N^{QM}, p_N^*) < \alpha(\mathcal{W}_N^{SM}, p_N^*)$$

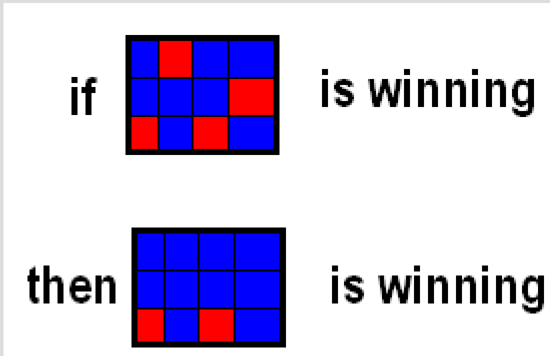
OUTLINE OF THE COURSE

BINARY DICHOTOMOUS VOTING RULES

SIMPLEST VOTING SITUATION



Monotonicity



Unanimous YES

Absence of YES

DICHOTOMOUS VOTING RULES

- **BINARY RULES**

$$S = (S^Y, S^N)$$

- **TERNARY RULES**

$$S = (S^Y, S^A, S^N)$$
$$S = (S^Y, S^H, S^N)$$

- **QUATERNARY RULES**

$$S = (S^Y, S^A, S^H, S^N)$$

NOTATION

N = Set of potential voters

S^N = Set of those who vote **n**o

S^H = Set of those who stay at **h**ome

S^A = Set of those who come and **a**bstain

S^Y = Set of those who vote **y**es

n = total number of potential voters

s^N = number of those who vote **n**o

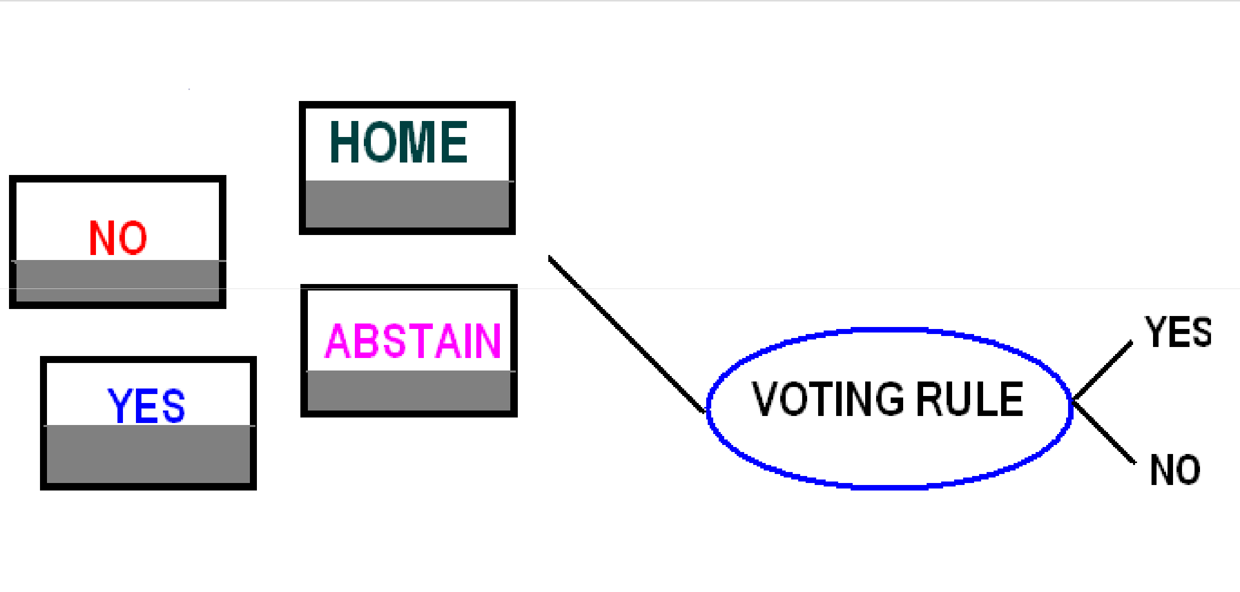
s^H = number of those who stay at **h**ome

s^A = number of those who come and **a**bstain

s^Y = number of those who vote **y**es

QUATERNARY VOTING RULES

NOT THAT SIMPLEST VOTING SITUATIONS



DICHOTOMOUS

RESULT

$$\mathcal{W} = \{S : S \text{ leads to the acceptance of the proposal}\}$$

DIFFERENCE BETWEEN BINARY AND OTHERS

INCENTIVES TO VOTE NON SINCERELY

- No binary rule is **manipulable**: voters who are in favor of the proposal have no incentive to vote no, voters who are against the proposal have no incentive to vote yes
- **This does not hold any more with ternary or quaternary voting rule**. Example: when there is a participation quorum a voter may be better by staying home than showing up and voting no.

OUTLINE OF THE COURSE

I. Introduction

II. Binary voting rules

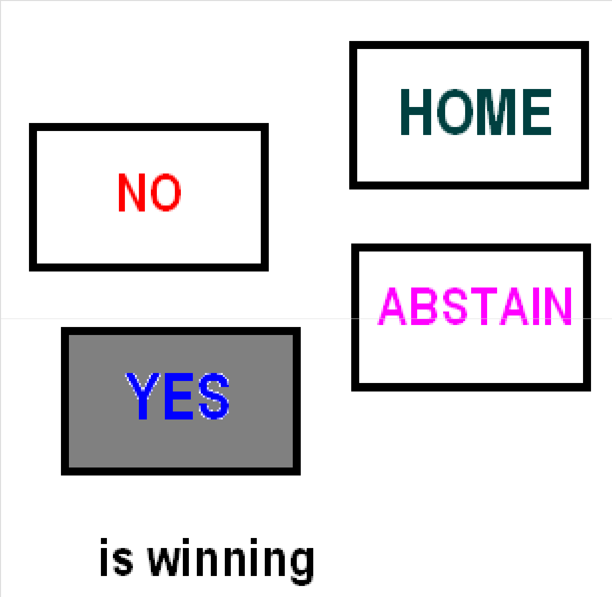
III. Ternary and quaternary voting rules

A. Definition - Properties

B. Examples: Majorities with quorum

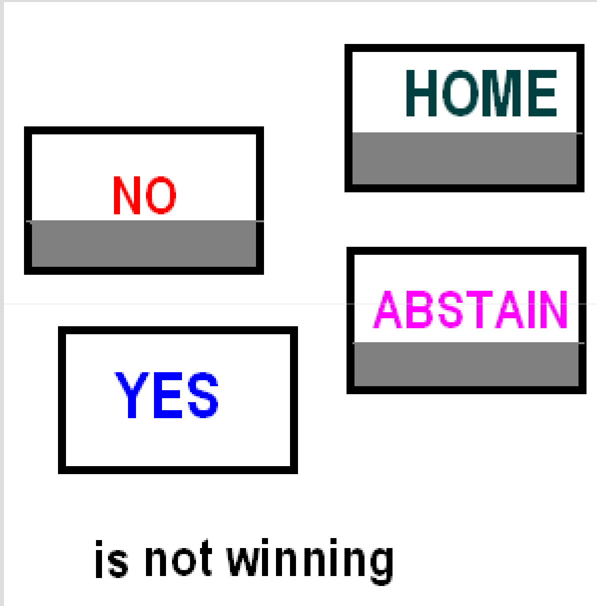
QUATERNARY VOTING RULE: PROPERTIES

Unanimous YES



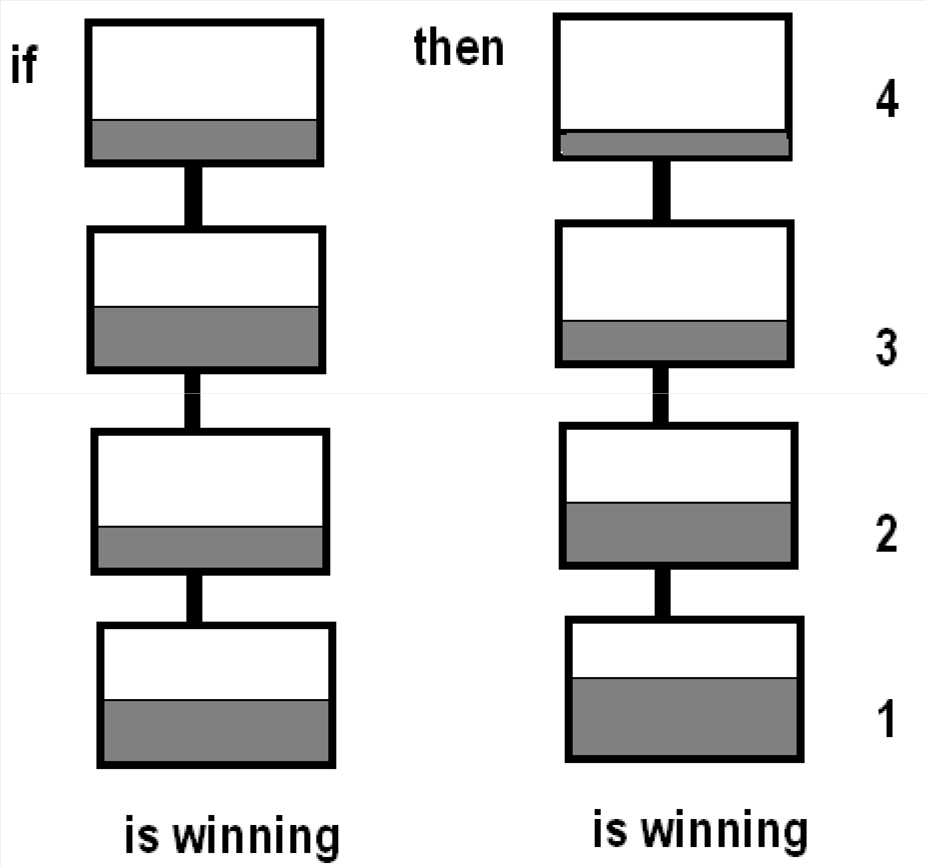
If **all voters** vote **yes** the result should be **yes**

Absence of YES



If **no voter** votes **yes** the result should be **no**

MONOTONOCITY FOR ORDERED OPTIONS



- If the options (yes, abstain, home and no) can be ordered in terms of support for yes, more support should be in favor of a final yes

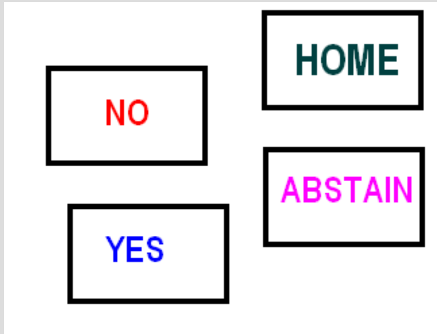
QUATERNARY VOTING RULE ARE NOT ORDERED

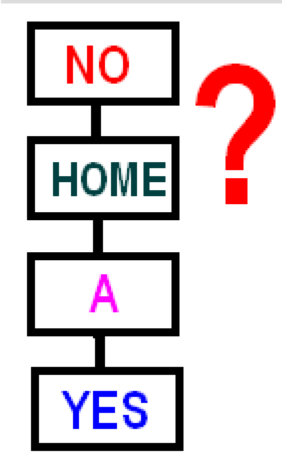
Example: Belgian Parliament (n=150)

simple majority: $s^Y > s^N$

with a participation quorum

$s^Y + s^A + s^N > n/2$



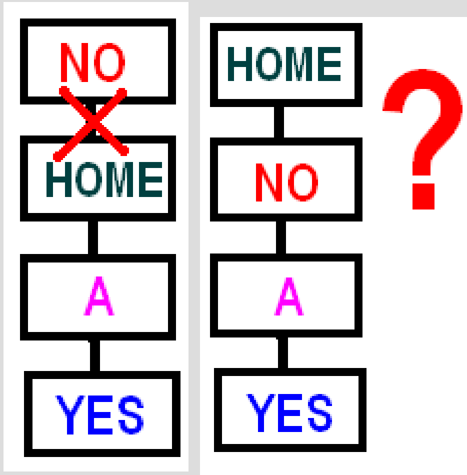


$s^N = 40,\ t^N = 20$

$s^H = 60,\ t^H = 80$

$s^A = 0,\ t^A = 0$

$s^Y = 50,\ t^Y = 50$

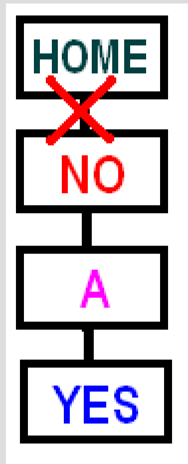


$s^H = 60,\ t^H = 40$

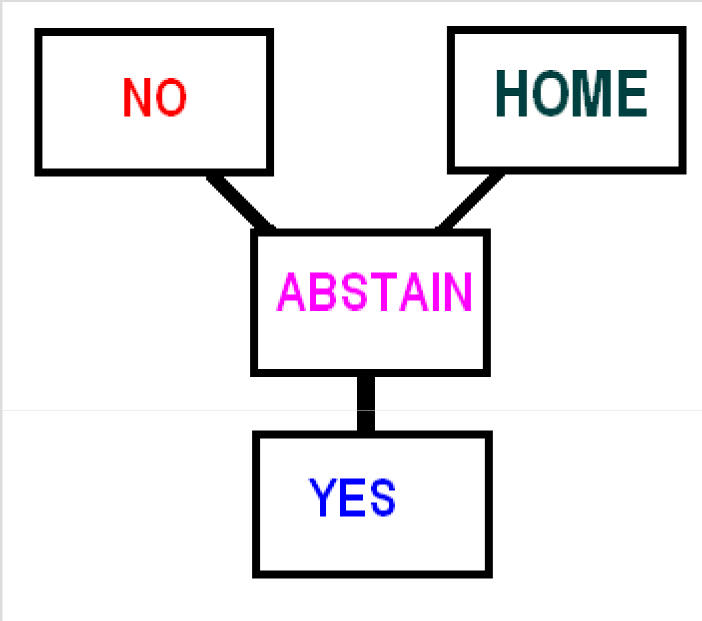
$s^N = 40,\ t^N = 60$

$s^A = 0,\ t^A = 0$

$s^Y = 50,\ t^Y = 50$



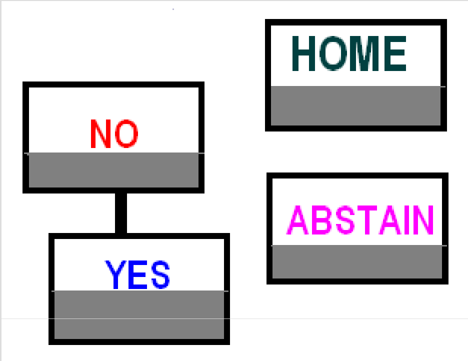
MONOTONICITIES OF THE BELGIAN PARLIAMENT:



Simple majority with a participation quorum

QUATERNARY RULES: MONOTONICITIES

NY



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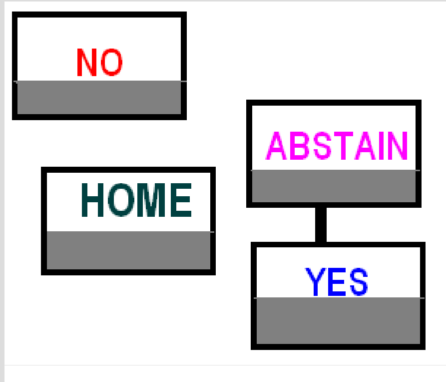

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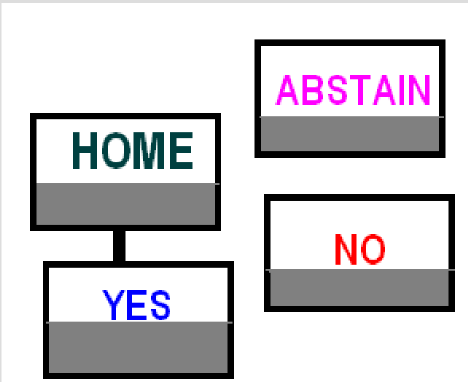
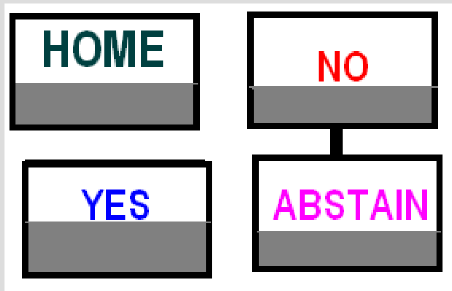
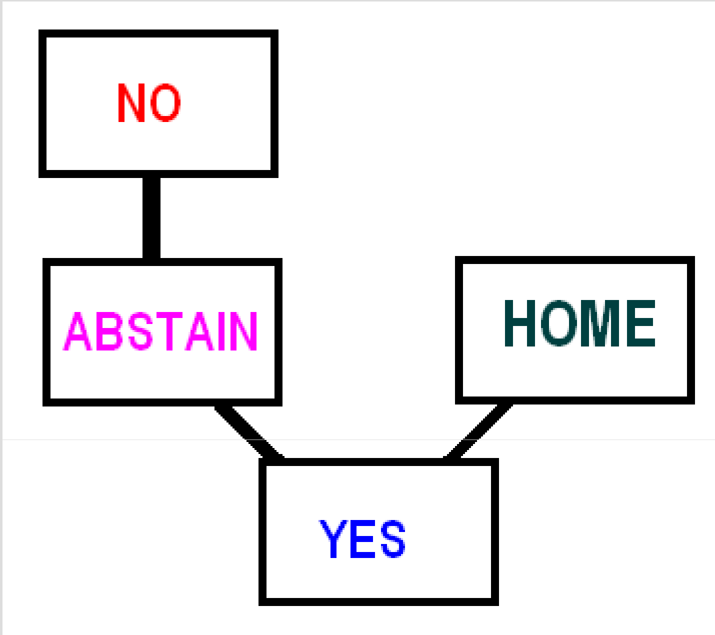


NA



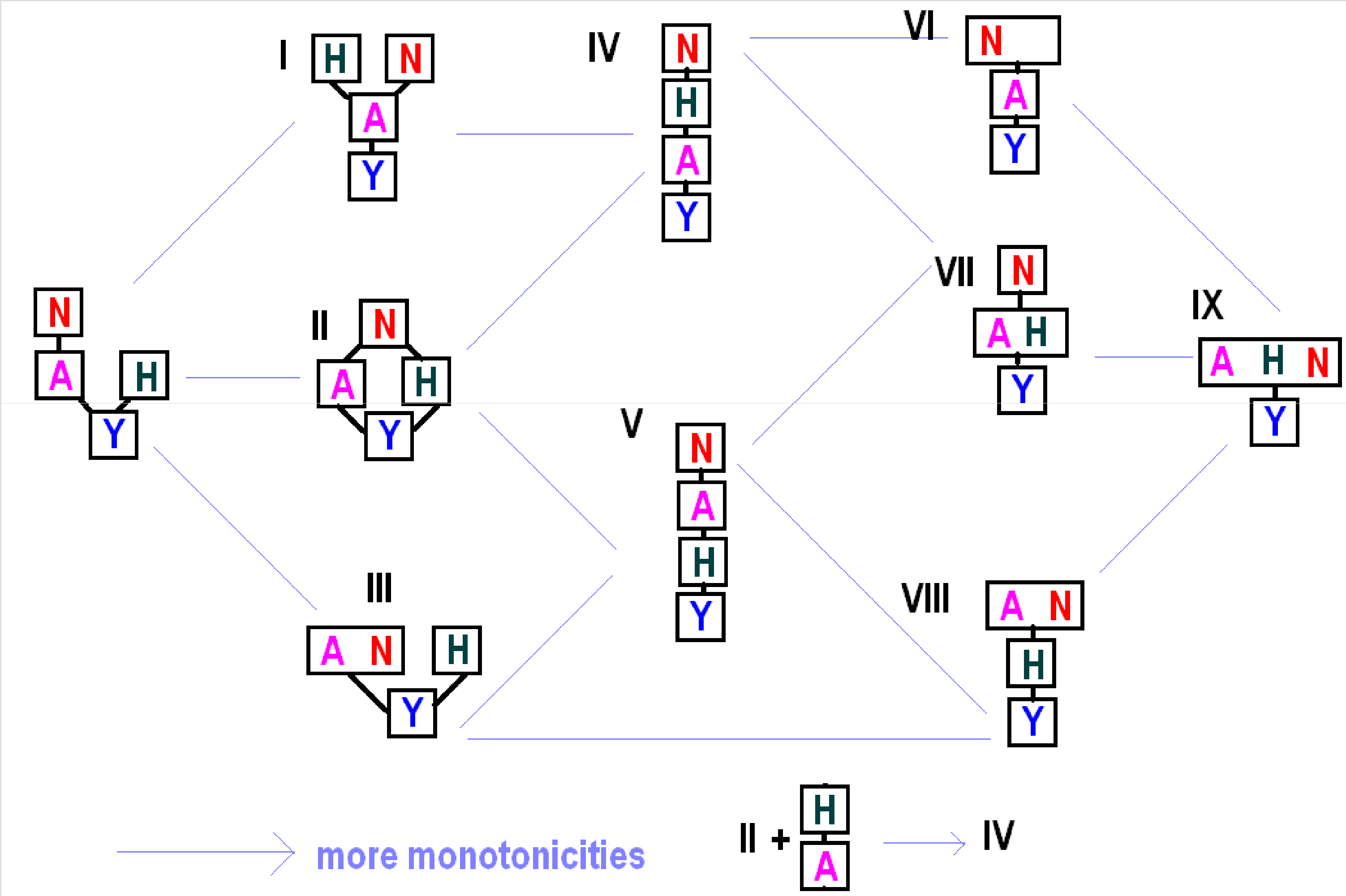
NA + AY imply NY

MINIMAL MONOTONICITIES



A QUATERNARY DICHOTOMOUS VOTING RULE SATISFIES AT LEAST THESE MINIMAL MONOTONICITIES

MORE MONOTONICITIES



OUTLINE OF THE COURSE

I. Introduction

II. Binary voting rules

III. Ternary and quaternary voting rules

A. Definition - Properties

B. Examples: Majorities with quorum

MAJORITIES AND QUORUM IN PARLIAMENT

For $\frac{1}{2}<q<1$

- Absolute majority $s^Y>q\,n$
- Simple majority $s^Y> q\,(s^Y+s^N)$
- Majority of present voters $s^Y> q\,(s^Y+s^A+s^N)$

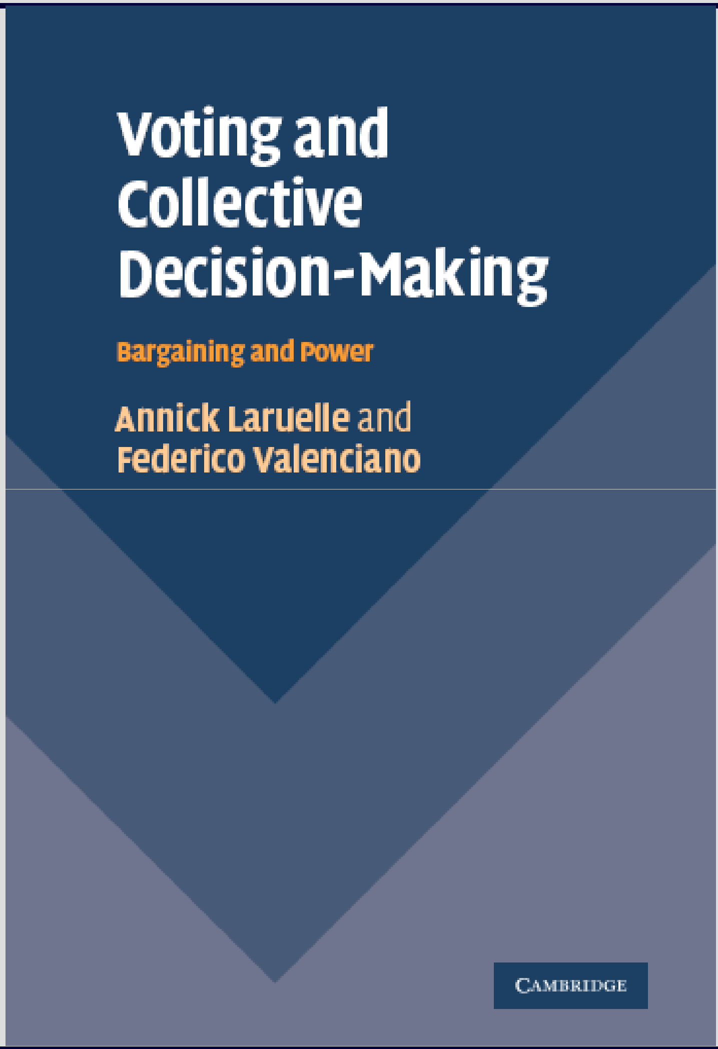
For $k<q$

- Approval quorum $s^Y> k\,n$
- Participation quorum $s^Y+s^A+s^N>kn$

SOME EXAMPLES

- The Swedish Riksdag uses a 1/2-simple majority
- The Finish parliament uses a 1/2-majority of present voters
- The Estonian parliament uses a absolute 1/2-majority
- The rule used for referendum in Germany is a 1/2-simple majority with an 1/4-approval quorum
- The Belgian Chamber of Representatives uses a 1/2-simple majority with a 1/2-participation quorum.

THIS PRESENTATION IS BASED ON



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- 2010, Egalitarianism and utilitarianism in committees of representatives, Social Choice and Welfare 35(2), 221-243. Joint with Federico Valenciano.
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- 2012, Quaternary dichotomous voting rules, Social Choice and Welfare 38, 431-454. Joint with Federico Valenciano.
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