# **Party-list Proportional Representation (PLPR) in Partisan Politics:** A New Voting Paradox Hayrullah Dindar (with Gilbert Laffond and Jean Lainé) hayrullah.dindar@bilgi.edu.tr



We show that the outcome of indirect elections under PLPR, where party platforms are endogeneously determined by their supporters preferences, may lead to an outcome that is Pareto-dominated by the direct elections outcome even when no group of supporters of one party has an incentive to leave that party and join another one.

## **Party-list Proportional Representation**

- Representative democracy combines two levels of choice:
- 1) How citizens' votes are transformed into assemblies.
- 2) How decisions are made within assemblies.
- Party-list proportional representation (PLPR) works as follows: 1) Political parties design a list involving as many candidates as positions to be filled.

# **Endogenous political supply, does the** inconsistency prevail?

- Our analysis is based on a model where the platform of each party emanates from the preferences of its supporters. Numerous examples support this approach:
- 1)In both US and French primary presidential elections, parties running for office design their programs according to their partisan's views.
- 2)In Turkey, the recent evolution of the leading party AKP regarding the Kurdish Peace Process may be attributed to voters taking "more nationalistic" positions

## **Main Features of Our Model**

• Each voter is characterized by an ideal platform: vector of YESor-NO positions regarding a given number of *mutually indepen*-



- Call balanced a consistent partisan map where merging or splitting parties either increases the distance between one partisan's ideal and her party platform, or increases the distance between the direct and the indirect outcome.
- Studying the structure of balanced maps may be interesting.

#### **Domain restrictions to avoid the paradox?**

- Which domain restrictions makes partisan maps immune to the PLPR paradox? An easy answer relates to the size of issue-wise majority margins.
- **Proposition 4:** Suppose that in the voter profile  $x^N$ , for each issue majority is strictly more than 75%. Then no partisan map  $(x^N, \mathcal{S})$ faces the PLPR paradox (strategic or preferential).



- 2) Citizens vote for a single party.
- 3) Each of the parties gets a proportion of seats equal to its vote share (ignoring any rounding issue).
- It is often claimed that *PR* is the best device in terms of *faithful* accounting of the citizenry wills, which can be given two meanings:
- 1) The ability of assemblies to represent the full diversity of opinions within a nation:
- a) "The system of proportional representation ensures that virtually every constituency in the country will have a hearing in the national and provincial legislatures." Bishop Desmond Tutu, The Rainbow People of God (1994)
- b) "... the portrait is excellent in proportion to its being a good likeness,...the legislature ought to be the most exact transcript of the whole society... the faithful echo of the voices of the people." (James Wilson, at the Constitutional Convention)
- 2) Assembly decisions can be regarded as the decisions of the nation itself, thus result of representative democracy should be identical (or close enough) to result of direct democracy:
- a) "Representative democracy is at best a working model of direct democracy and is most successful when it generates decisions as close as possible to those that would be generated in a direct democracy. [...] It is direct democracy (actual or ideal) that is used as a measuring rod" (Chamberlin and Courant, 1983)

# **Pro and Con Arguments -PLPR**

• Two-party polities less probable, higher voter turnout. Overcome gerrymandering, malapportionment and pork barrelling.

- dent issues.
- Party platforms are multichotomous, designed by issue-wise majority voting within the set of partisans.
- Each voter ranks platforms by means of the symmetric (Hamming) distance: distance=  $\ddagger$  of disagreements,
- and votes for the party with the platform closest to her ideal.
- Given the resulting set of party platforms, parliamentary seats are distributed proportional to vote shares.
- The final outcome is determined by issue-wise majority from the parliamentary profile.



### **Related Literature**

#### **Referendum Paradox**

• The PLPR paradox can be interpreted as a strong version of the referendum paradox (Nurmi, 1998, 1999).

• Example: 15 voters, 5 districts $\Rightarrow$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
	1	1	0	0	0
	1	0	1	0	0
	0	1	1	0	0

- In our multichotomous choice framework, parties can be interpreted as districts
- Preferential (resp. strategic) stability pertains to the impossibility for a group of citizens in some district to achieve a better representation (a more preferred final outcome) by jointly moving to another district.
- Hence, the PLPR paradox states that districting can be designed so as to secure an accurate representation of the citizenry and yet bring a Pareto-dominated outcome.

### Studies on direct election outcome with multiple issues

- Several studies on multiple referendum consider the properties of the direct election outcome (DEO).
- Non-separable preferences  $\Rightarrow$  (*DEO*) may be Pareto dominated :Lagerspetz (1996), Lacy and Niou (2000) and Brams, Kilgour and Zwicker (1997).
- Separable preferences  $\Rightarrow$  (*DEO*) may be Pareto dominated :Sanver and Özkal-Sanver (2006)
- Hamming distance criterion  $\Rightarrow$  (*DEO*) Pareto optimal: Cuhadaroglu and Lainé (2012)
- The PLPR paradox takes (DEO) as "measuring rod" and states that pure proportional representation fails at achieving this outcome even when party platforms give an accurate view of the citizenry.



Senate election district of South Essex

- Makes "a majority for a single party" less probable. May cause political instability and cause anticipated elections to absorb the attention of politicians.
- Endow small parties with too much bargaining power in the search for a government coalition.



#### **Definitions**

- A partisan map is called **consistent** when no partisan of some party votes for another party.
- We call preferentially stable a partisan map  $(x^N, \mathcal{S})$  where no subset of supporters of a party can by jointly supporting another party, contribute to this new party platform that is closer to their ideals.
- *Preferentially stability* ignores the effect of voters' moves on the final outcome. Getting closer to party platform may be at the cost of a less preferred final outcome.
- We call strategically stable a partisan map where no subset of partisans of one party can by joining a new party get a final outcome that they all prefer to the initial outcome.
- A *preferentially* and *strategically* stable partisan map faithfully accounts for the diversity of opinions.

## **MAIN RESULT**

- **Proposition 1:** Our main finding is that the final outcome of a preferentially and strategically stable partisan map, may be unanimously less preferred to the outcome of direct elections.
- Put differently, the outcome of *PLPR*, in the most favorable case of endogenous party platforms, may be Pareto-dominated by the direct democracy outcome.
- This defines a new voting paradox, called the *PLPR* paradox.

### **Further Remarks**

#### **Ostrogorski and Anscombe's Paradoxes**

- The PLPR paradox is related to the Ostrogorski paradox: -There are two parties, 1 and 2, competing over multichotomous platforms. -1 takes the view of the electoral majority on every issue (2 takes the minority view).
- -2 wins the election by a clear-cut majority: Daudt and Rae (1976), Kelly (1989), Laffond and Lainé (2006), (2009)
- The Ostrogorski paradox is close in spirit (but not equivalent) to the Anscombe's paradox: Anscombe (1976), Wagner (1983), (1984), Laffond and Lainé (2013).
- A major difference between the Ostrogorski paradox and the PLPR paradox deal with the number and the endogeneity of party platforms.
- Moreover, we are not focusing on the winning party but instead, on the discrepancy between indirect/direct democracy outcomes.

### **Multiple elections paradox**

- The PLPR paradox also relates to the multiple elections paradox: Brams, Kilgour and Zwicker (1998), Scarsini (1998).
- Defined as the situation in which the direct election outcome coincides with the fewest ideals (maybe none) or is tied for the fewest.
- We show that the PLPR system may also have the same default.



# **Inconsistency between PR and direct** elections (fixed political supply)

- Three possible decisions a, b, c and three political parties 1, 2, 3.
- Party preferences are linear orders:  $a \succ_1 b \succ_1 c, b \succ_2 c \succ_2 a$ ,  $c \succ_3 a \succ_3 b.$
- Voters rank parties lexicographically:



• Vote shares leads to majority cycle in parliament:  $a \rightarrow b \rightarrow c \rightarrow$ a.

• By contrast, citizens' preferences: b is the Condorcet winner.

#### **Optimal Partisan Maps**

- Preferential stability aims at describing a platform set that best represents the diversity of voters opinions.
- An alternative way to capture the same idea is to consider partisan maps with q parties where the platform set is at minimal distance to voters profile.
- An *optimal* partial map need not be *preferentially stable* and visa versa
- **Proposition 2:** The PLPR paradox may also hold at an *optimal* and *strategically* stable partisan map.

#### **Number of parties?**

**Proposition 3:** For any *consistent* political landscape with only two parties (with any number of issues), the PLPR paradox occurs only if ties are broken in a specific way.

- As many parties as different ideals  $\Rightarrow$  no paradox.
- Similarly, a unique party  $\Rightarrow$  no paradox.

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