## Motivation

It is proposed a model of a legislature, formed by several parties, which have to vote for or against a certain bill in the presence of a lobbyist who is interested in a certain outcome of the vote. We show that the easiness to manipulate a legislature decision by the lobbyist is increasing with the number of parties. A high electoral threshold leads to fewer parties represented, and consequently, decreases the easiness to change a legislature decision by the lobbyist. On the other hand, high electoral threshold may cause a misrepresentation of voters. We show that if the threshold is higher that $6 \%$, the impact of the misrepresentation effect becomes significant.


## Legislature

- $n$ - number of parties in a parliament.
- Each party has a certain position about a bill under voting: $D_{i} \in[-1 ; 1], i=1, \ldots, n$, such that $D_{i}=2 X_{i}-1$, where $X_{i} \in[0 ; 1], X_{i} \sim \operatorname{Beta}(\alpha, \alpha), \alpha$ is a given parameter, so that as $\alpha \rightarrow \infty$, then the political system is less ideologically polarized.
- Each party controls a certain number of seats, which defines its' voting weight $w_{i} \sim \operatorname{Poiss}(\lambda)$ for each $i \in\{1, \ldots, n\}$.
- Simple majority voting rule and .


## Lobbyist

- The lobbyist offers to each party a nonnegative payment $P_{i} \in[0 ; 1]$ for voting for the bill, and every party accepts the offer if $P_{i}-D_{i} \geq 0$
- L minimises the total amount of payment to parties, taking into account total weights of bribed parties:

$$
T(n)=\min \left\{\sum_{i=1}^{n} P_{i}\right\}, \text { s.t. } \sum_{P_{i}-D_{i}} w_{i}>\frac{\sum_{j=1}^{n} w_{j}}{2}
$$

- The ease of manipulation is the expected $T(n)$, denoted as $E_{T}(n)$


Number of parties
$\qquad$ 1 Ease of
manipulation

Electoral threshold would decrease the number of parties

## Misrepresentation

- Random number of parties compete in elections: $n^{\prime} \sim \operatorname{Poiss}\left(\lambda_{p}\right)$.
- Each party has a random position about the bill: $D^{\prime}{ }_{i} \in[-1 ; 1]$.
- Each party obtains random number of votes: $v_{i} \sim \operatorname{Poiss}\left(\lambda^{*}\right)$.
- Popular preference $\Pi=\{A, R, I\}$ about the bill ("Accept", "Reject", or "Indifferent") is based on $v_{i}$ and $D^{\prime}{ }_{i}$.
- There is an electoral threshold $t$, so that some parties do not pass it and therefore, do not get any seat in legislature.


## Lobbyist

- The lobbyist has random standing about the bill: $D_{L} \in[-1 ; 1]$.
- He offers to each party a nonnegative payment $P_{i} \in[0 ; 1]$ for voting for the bill, and every party accepts the offer if $P_{i}-D_{i} \geq 0$.
- If the lobbyist succeed to bribe parties, the legislature adopts the decision $\Pi_{l}(t)=\{A, R, I\}$, desired by the lobbyist.
- If the lobbyist cannot bribe the necessary parties, the legislature the legislature adopts the truthful decision, based on $D_{i}$ and $w_{i}$.

