The Problem of the Divided Majority

Preference Aggregation and Uncertainty

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The Divided Majority
Three Candidates: Red, Blue and Green

Electorate (group, committee, state, etc.) is characterized by the following preference profile

<table>
<thead>
<tr>
<th>Type of Voter</th>
<th># Voters</th>
<th>Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grues</td>
<td>2</td>
<td>Green ≻ Blue ≻ Red</td>
</tr>
<tr>
<td>Reds</td>
<td>3</td>
<td>Red ≻ Blue ≈ Green</td>
</tr>
<tr>
<td>Bleens</td>
<td>2</td>
<td>Blue ≻ Green ≻ Red</td>
</tr>
</tbody>
</table>

Reds voters constitute a weak majority

Red is the worst outcome for an absolute majority of voters

Coordination Problem: Grues and Bleens can avoid the ‘bad’ outcome if they coordinate
The Divided Majority

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- Central to the analysis of electoral systems since at least Jean Charles de Borda (1781), Marie Jean Nicolas Caritat Marquis de Condorcet (1785)

- **Condorcet-Winner (Loser)** is defined as an alternative that can beat (that is beaten by) any other alternative in pairwise comparison:

  - 4 voters prefer **Green** over **Red**, 4 voters prefer **Blue** over **Red**, **Red** is a Condorcet-Loser

- Infamous real world examples exist...
Central to the analysis of electoral systems since at least Jean Charles de Borda (1781), Marie Jean Nicolas Caritat Marquis de Condorcet (1785)

Condorcet-Winner (Loser) is defined as an alternative that can beat (that is beaten by) any other alternative in pairwise comparison:

◊ An absolute majority of voters prefer Gore over Bush and Nader over Bush, Bush is a Condorcet-Loser

Infamous real world examples exist... like the United States presidential election in Florida, 2000
Research questions

RQ1: Coordination Failures and Condorcet-Efficiency?

RQ2: Informational Structure?

RQ3: Individual level of sophistication?
Research questions

RQ1: Coordination Failures and Condorcet-Efficiency?

- Do multi-vote systems facilitate coordination in divided majority problems?
  Is coordination efficient, i.e., does coordination take place on the Condorcet-Winner?

RQ2: Informational Structure?

RQ3: Individual level of sophistication?
Research questions

RQ1: Coordination Failures and Condorcet-Efficiency?
▶ Do multi-vote systems facilitate coordination in divided majority problems? Is coordination efficient, i.e., does coordination take place on the Condorcet-Winner?

RQ2: Informational Structure?
▶ Do coordination failures increase if we consider more realistic situations with less information?

RQ3: Individual level of sophistication?
Research questions

RQ1: Coordination Failures and Condorcet-Efficiency?
- Do multi-vote systems facilitate coordination in divided majority problems?
  Is coordination efficient, i.e., does coordination take place on the Condorcet-Winner?

RQ2: Informational Structure?
- Do coordination failures increase if we consider more realistic situations with less information?

RQ3: Individual level of sophistication?
- How strategic do voters act?
  What is the impact of the underlying information structure on these results?
Why Lab experiments?

► Field Experiments:
  ◊ Offer invaluable data and evidence for the actual feasibility, and show that changes in voting methods alter the results, and that the methods are well accepted by voters (see Alós-Ferrer and Granić (2012), Baujard and Igersheim (2009) and Laslier and Van der Straeten (2008))
  ◊ Suffer from potential self-selection biases and lack of fully identifying participants’ preferences

► Laboratory Experiments:
  ◊ Controlled environment allows us to test certain properties that cannot be tested in the field
  ◊ Design of the experiment is based on Forsythe et al. (1993) and Forsythe et al. (1996)
  ◊ Experiments with single-peaked preferences and spatial representation: Dellis et al. (2010), Van der Straeten et al. (2010)
Design of the Experiment
336 participants in 12 sessions. The experiment follows a \( 3 \) (Voting method) \( \times 2 \) (Information structure) between subjects design.
Design

- 336 participants in 12 sessions. The experiment follows a 3 (Voting method) × 2 (Information structure) between subjects design.
- Voting methods:
  - **Approval Voting (AV):** Each voter can approve of as many alternatives as he/she likes. The alternative with the most approvals wins the election.
  - **Borda Count (BC):** Each voter distributes 3, 2, 1, and 0 points among the alternatives. The alternative with the most points wins.
  - **Plurality Voting (PV):** Each voter can cast one vote, a simple majority is enough to win the election.
336 participants in 12 sessions. The experiment follows a 3 (Voting method) × 2 (Information structure) between subjects design

**Voting methods:**
- Approval Voting (AV)
- Borda Count (BC)
- Plurality Voting (PV)

**Information structure:**
- **Full information** (FI): Participant know the payoffs (not the identities) of their group members
- **Incomplete information** (II): Participant know their own payoff only (more on this later)
Each session: 28 participants, randomly divided into 4 groups (7 participants each)

Each group participates in 8 elections with 4 available alternatives

Participants are informed about the election results and their corresponding payoffs

After 8 elections: randomly reassign the participants into 4 new groups and another series of 8 elections starts

Each participant plays 3 series of 8 elections (96 elections per session in total)

The experiment was conducted in the University of Konstanz’ own computer laboratory (Lakelab) using the computer software z-Tree (Fischbacher, 2007)
### Induced Preference Profile

<table>
<thead>
<tr>
<th>Number of Participants</th>
<th>Payoffs in ECU</th>
<th>Induced Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
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**Condorcet-Winner and Condorcet-Loser**

- $D$ is the unique Condorcet-Winner, it beats every other alternative in a pairwise comparison.
- $B$ is the unique Condorcet-Loser, it loses against every other alternative in a pairwise comparison.
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- **Condorcet-Winner** and **Condorcet-Loser**
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► In light of RQ1:

◊ Coordination failures arise if $B$ wins an election, $B$ should win less often under AV and BC than under PV

◊ Coordination should take place on the Condorcet-Efficient alternative $D$
Results
Aggregate Data: Election Outcomes

Fraction of won Election

<table>
<thead>
<tr>
<th>Treatment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVII</td>
<td>9%</td>
<td>19%</td>
<td>25%</td>
<td>47%</td>
</tr>
<tr>
<td>PVFI</td>
<td>4%</td>
<td>7%</td>
<td>35%</td>
<td>54%</td>
</tr>
<tr>
<td>BCII</td>
<td>5%</td>
<td>9%</td>
<td>14%</td>
<td>72%</td>
</tr>
<tr>
<td>BCFI</td>
<td>7%</td>
<td>7%</td>
<td>13%</td>
<td>73%</td>
</tr>
<tr>
<td>AVII</td>
<td>8%</td>
<td>5%</td>
<td>24%</td>
<td>62%</td>
</tr>
<tr>
<td>AVFI</td>
<td>6%</td>
<td>2%</td>
<td>17%</td>
<td>75%</td>
</tr>
</tbody>
</table>
Aggregate Data: Coordination Failures

The Problem of the Divided Majority – p. 14
Aggregate Data: Condorcet Efficiency

The Problem of the Divided Majority – p. 15
Aggregate Data: AV

(a) AVFI

(b) AVII
Aggregate Data: BC

(c) BCFI

(d) BCII
Aggregate Data: PV

(e) PVFI

(f) PVII
Ties, Close Races, Duverger’s Law

<table>
<thead>
<tr>
<th></th>
<th>No Ties</th>
<th>Two-Way Ties</th>
<th>Three-Way Tie</th>
<th>Four-Way Tie</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVFI</td>
<td>139</td>
<td>39</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>AVII</td>
<td>124</td>
<td>45</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>BCFI</td>
<td>159</td>
<td>20</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>BCII</td>
<td>159</td>
<td>27</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>PVFI</td>
<td>118</td>
<td>38</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>PVII</td>
<td>132</td>
<td>55</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

- AV creates more ties than BC and PV (Kruskal-Wallis, weakly significant for FI, p-value=0.082, highly significant for NI, p-value=0.001)
- Change from FI to II increases Ties for AV (WRS, p-value=0.087)
Ties, Close Races, Duverger’s Law

The Problem of the Divided Majority – p. 20

The chart above illustrates the share of votes for different treatments under various rankings in a divided majority scenario. The treatments include PVII, PVFI, BCII, BCFI, AVII, and AVFI. The chart shows the share of votes for each treatment under different ranking scenarios:

- **Rank 1**
- **Rank 1 P5**
- **Rank 1+2**
- **Rank 1+2 P5**

The treatments and their corresponding share of votes are as follows:

- **PVII**
  - Rank 1: 87%
  - Rank 1 P5: 51%
  - Rank 1+2: 83%
  - Rank 1+2 P5: 48%

- **PVFI**
  - Rank 1: 91%
  - Rank 1 P5: 53%
  - Rank 1+2: 85%
  - Rank 1+2 P5: 50%

- **BCII**
  - Rank 1: 57%
  - Rank 1 P5: 31%
  - Rank 1+2: 58%
  - Rank 1+2 P5: 32%

- **BCFI**
  - Rank 1: 58%
  - Rank 1 P5: 33%
  - Rank 1+2: 59%
  - Rank 1+2 P5: 33%

- **AVII**
  - Rank 1: 60%
  - Rank 1 P5: 33%
  - Rank 1+2: 60%
  - Rank 1+2 P5: 34%

- **AVFI**
  - Rank 1: 64%
  - Rank 1 P5: 37%
  - Rank 1+2: 64%
  - Rank 1+2 P5: 37%

These results highlight the impact of different ranking systems on the share of votes in a divided majority scenario.
Individual Voting Behaviour

- AV does not degenerate to PV: irrespective of information treatment, average approvals » 1

- Strategic voting:
  - Under FI, fraction of sincere ballots cast under AV: 83.26%. Under PV: 51.30%. Under BC: 41.96%
  - Under NI, fraction of sincere ballots cast under AV: 93.01%. Under PV: 75.82%. Under BC: 46.5%

- No impact on information structure on sincere voting for AV and BC. As in other studies, under PV and uncertainty sincerity increases
Conclusion

- Multi-votes methods (‘One Man, many Votes’) like AV and BC facilitate coordination among the divided majority groups.

- Coordination failures are not only reduced effectively, multi-votes methods also increase coordination efficiently as indicated by the corresponding large winning frequencies of the Condorcet-Winner.

- Coordination on the Condorcet-Winner is much harder to establish under a single-vote method than under a multiple-vote method. The limited amount of information that is transmitted through a Plurality Voting ballot hinders coordination.

- Informational structure (i.e., responsiveness towards it) may serve as another dimension to evaluate the merits of voting methods.
Thank you for your attention
0.1 Bibliography


Approval Voting

- Approval Voting (AV): Proposed by Steven J. Brams and Peter C. Fishburn (1977)

- Each voter can assign 1 or 0 votes to each candidate. That is, “approve of” as many candidates as wished. The candidate with the most approvals wins

- Arguments in the literature: AV provides an accurate reflection of voters’ wishes and is not vulnerable to voter manipulation (see Brams and Fishburn, 1978; Fishburn, 1978a,b; Brams and Fishburn, 2005; Wolitzky, 2009)
Preliminary Work: Field Experiments

- Get permission from State and Federal Authorities *This was funny.*
- Inform all involved registered voters per mail prior to the election, explain the method. *This was expensive*

Election day: established one experimental polling station in each of the preselected constituencies (same building, different room). *This was a lot of work*

Use official ballot boxes and voting urns.

After casting a ballot in the official polling stations, a “certificate“ was handed over to the voters by the polling clerks which qualified them for participation in the experiment.

Guarantees undisturbed official election and that we only got actual voters; but allows for a serious drop-off and maybe self-selection effects.
2008 State election in Hesse

1909 eligible voters went to the polls, of which, in turn, 967 participated in our experiment (participation rate 50.7%). With 6 invalid votes, the data set consists of 961 AV ballots.
## 2008 State election in Hesse

<table>
<thead>
<tr>
<th>Party</th>
<th>Approvals</th>
<th>AV Rank</th>
<th>Official Votes</th>
<th>PV Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPD</td>
<td>53.8%</td>
<td>1</td>
<td>38.9%</td>
<td>1</td>
</tr>
<tr>
<td>CDU</td>
<td>44.6%</td>
<td>2</td>
<td>36.0%</td>
<td>2</td>
</tr>
<tr>
<td>The Greens</td>
<td>36.1%</td>
<td>3</td>
<td>7.0%</td>
<td>4</td>
</tr>
<tr>
<td>FDP</td>
<td>32.6%</td>
<td>4</td>
<td>9.0%</td>
<td>3</td>
</tr>
<tr>
<td>The Left</td>
<td>12.3%</td>
<td>5</td>
<td>4.9%</td>
<td>5</td>
</tr>
<tr>
<td>Animal Protection Party</td>
<td>9.6%</td>
<td>6</td>
<td>0.8%</td>
<td>7</td>
</tr>
<tr>
<td>The Family Party</td>
<td>9.6%</td>
<td>6</td>
<td>0.2%</td>
<td>12</td>
</tr>
<tr>
<td>The Free Voters</td>
<td>7.1%</td>
<td>8</td>
<td>0.5%</td>
<td>9</td>
</tr>
<tr>
<td>The Republicans</td>
<td>3.3%</td>
<td>9</td>
<td>1.0%</td>
<td>6</td>
</tr>
<tr>
<td>The Popular Vote</td>
<td>2.9%</td>
<td>10</td>
<td>0.2%</td>
<td>13</td>
</tr>
<tr>
<td>NPD</td>
<td>2.8%</td>
<td>11</td>
<td>0.8%</td>
<td>7</td>
</tr>
<tr>
<td>The Hessian Pirates</td>
<td>2.8%</td>
<td>11</td>
<td>0.3%</td>
<td>10</td>
</tr>
<tr>
<td>The Grey Party</td>
<td>2.5%</td>
<td>13</td>
<td>0.2%</td>
<td>13</td>
</tr>
<tr>
<td>UB</td>
<td>2.1%</td>
<td>14</td>
<td>0.1%</td>
<td>15</td>
</tr>
<tr>
<td>The Violet Party</td>
<td>1.0%</td>
<td>15</td>
<td>0.3%</td>
<td>11</td>
</tr>
<tr>
<td>PSG</td>
<td>0.9%</td>
<td>16</td>
<td>0.1%</td>
<td>15</td>
</tr>
<tr>
<td>Civil Liberties Party</td>
<td>0.9%</td>
<td>16</td>
<td>0.1%</td>
<td>15</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>225.0%</strong></td>
<td></td>
<td><strong>100.0%</strong></td>
<td></td>
</tr>
</tbody>
</table>